BFFT Formalism Applied to the Minimal Chiral Schwinger Model

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Abstract

We consider the minimal chiral Schwinger model, by embedding the gauge noninvariant formulation into a gauge theory following the Batalin-Fradkin-Fradkina-Tyutin point of view. Within the BFFT procedure, the second class constraints are converted into strongly involutive first-class ones, leading to an extended gauge invariant formulation. We also show that, like the standard chiral model, in the minimal chiral model the Wess-Zumino action can be obtained by performing a q-number gauge transformation into the effective gauge noninvariant action.

1 Introduction

Following Dirac's conjecture a critical issue in the study of a gauge model is the presence of first-class constraints [1]. First-class constraints are related to symmetries while the second-class ones may imply some ambiguities when treated as quantum field operators. The physical status of a theory is chosen by imposing complementary conditions which are given by the first-class constraints. The presence of second-class constraints is in general avoided. There are many procedures which allow the exclusion of these constraints from the effective action [2]. One of them is the Batalin-Fradkin-Fradkina-Tyutin (BFFT) method [3, 4, 5], which converts second-class constraints into first-class ones by introducing auxiliary fields. The BFFT formalism has been employed in different models, as for example, the chiral boson model [6], the massive Maxwell and Yang-Mills theories [7, 8, 9], the CP^{N-1} model [10], the non-linear sigma model [11], the chiral Schwinger model [12, 13] and more recently a fluid field theory [14]. As expected, the implementation of the BFFT method through the introduction of new fields gives rise to a kind of a Wess-Zumino term which turns the resulting extended theory gauge invariant. In particular, an elegant way of obtaining the Wess-Zumino term and the effective action is the BRST-BFV procedure [15].

On the other side, two dimensional models have played an important role in theoretical physics as a laboratory where many interesting phenomena can be studied in a fashion which is usually easier to handle than more realistic four dimensional theories [16]. One well known model is the two dimensional quantum electrodynamics (QED_2) which was introduced long ago by Schwinger [17] to discuss dynamical mass generation for gauge fields without breaking the gauge symmetry [18]. More recently, Jackiw and Rajaraman proposed a model [19] with a chiral coupling between the two dimensional gauge and fermion fields mimicking the weak interactions of the standard model. This two dimensional model, known as the chiral Schwinger model, happens to be gauge anomalous although unitary and carries an arbitrary regularization parameter a in all of its physical quantities (mass, propagator, etc) (see, *e. g.*, [20, 21] and

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references therein). Another interesting two dimensional chiral model is the one that describes right or left movers, *i. e.*, chiral bosons which were introduced by Siegel [22] inspired on the heterotic string and later reobtained by Floreanini and Jackiw [23]. In particular, Harada [24, 25] considered a version of the chiral Schwinger model but without the right-handed fermions. He showed that this model known as the minimal chiral Schwinger model corresponds to a gauged version of the Floreanini-Jackiw chiral boson. Naturally, this model share some properties with the complete chiral model as the gauge anomaly and the dependence on an arbitrary parameter (which is usually called a, as in the original Jackiw-Rajaraman model) and the novelty here is the description of chiral bosons which in some sense represents the motion of superstrings. Furthermore, a left-handed Wess-Zumino (WZ) action has been built for this model by considering an antichiral constraint [26, 27, 28].

In this paper we discuss the Hamiltonian formalism for the minimal chiral Schwinger model using the BFFT method. The application of this method introduces in a very natural way the chiral constraints in the model. Otherwise, one would be forced to put these constraints by hand. Then, using the BRST-BFV procedure we obtain the Wess-Zumino term with a set of the first-class constraints and the total effective action.

We also show that, like the standard chiral model, in the minimal chiral model the WZ action can be obtained by performing a q-number gauge transformation into the effective gauge non-invariant (GNI) action.

This paper is organized as follows: In section II we discuss the conversion of the second-class to first-class constraints, by using the BFFT method for the miminal chiral Schwinger model. We obtain the corresponding extended Hamiltonian in strong involution with the first class constraints. In section III, we obtain the extended gauge invariant effective action which brings in a Wess-Zumino term. In section IV we discuss the generation of the WZ action by performing a q-number gauge transformation on the gauge non-invariant effective action. Finally, in section V we make some remarks about the "fermionization" of the extended gauge invariant formulation of the anomalous model for $a = 2$ and give general arguments contrary to the equivalence to the vector Schwinger model advocated in the literature [27, 29]. An appendix is also included where we present some details on the calculation of the extended canonical Hamiltonian.

2 Extended First-Class Hamiltonian

In order to implement the canonical BFFT scheme, it is necessary to specify the Hamiltonian together with the set of the constraints of the model [3, 4, 5]. Here we are going to apply the general BFFT method following the lines of work in Refs. [7, 8, 9].

To begin with, let us start by considering the bosonized version of the minimal chiral Schwinger model [24, 25], described by the following Lagrangian density [26, 27, 28]

$$\mathcal{L}[\phi, A_\mu] = \mathcal{L}_M[A_\mu] + \check{\mathcal{L}}[\phi, A_\mu]\,, \tag{2.1}$$

where the Maxwell Lagrangian is,

$$\mathcal{L}_M = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \tag{2.2}$$

the gauge noninvariant (GNI) contribution is given by,

$$\begin{aligned}\check{\mathcal{L}}[\phi, A_\mu] \quad &= \quad \dot{\phi}\,\phi' - (\phi')^2 + 2\,e\,\phi'(A_0 - A_1) - \frac{1}{2}\,e^2\,(A_0 - A_1)^2 \\ &\quad + \frac{1}{2}\,a\,e^2\left((A_0)^2 - (A_1)^2\right).\end{aligned} \tag{2.3}$$

with the Jackiw-Rajaraman parameter $a > 1$, overdot means partial time derivative ($\dot{\phi} = \partial_0\phi = \partial\phi/\partial t = \partial^0\phi$) and prime denotes partial space derivative ($\phi' = \partial_1\phi = \partial\phi/\partial x^1 = -\partial^1\phi$). The canonical momenta are given by

$$\Pi^\mu = \frac{\partial\mathcal{L}}{\partial\dot{A}_\mu} = F^{\mu 0}; \tag{2.4}$$

$$\Pi_\phi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = \phi'\,, \tag{2.5}$$

which imply the primary constraints

$$\Omega_1 = \Pi^0 \approx 0\,; \tag{2.6}$$
$$\Omega_2 = \Pi_\phi - \phi' \approx 0\,. \tag{2.7}$$

The corresponding canonical Hamiltonian is given by,

$$\begin{aligned} H_c &= \int \mathrm{d}x^1 \Big\{ \frac{1}{2}(\Pi^1)^2 + (\phi')^2 - 2\,e\,\phi'(A_0 - A_1) \\ &\quad + \frac{1}{2}\,e^2\,(A_0 - A_1)^2 - \frac{1}{2}\,a\;e^2\,(A_0)^2 \\ &\quad + \frac{1}{2}\,a\;e^2\,(A_1)^2 + \Pi^1\partial_1 A_0 \Big\}\,, \end{aligned} \tag{2.8}$$

where we are using the conventions $F^{10} = \Pi^1 = E^1 = \partial^1 A^0 - \partial^0 A^1 = \dot{A}_1 - \partial_1 A_0$. Time conserving of the primary constraints lead to the Gauss law

$$\begin{aligned} \Omega_3 &= \dot{\Omega}_1 = \{\Pi^0, H_c\} \\ &= \partial_1 \Pi^1 - e\,J^0 \approx 0\,, \end{aligned} \tag{2.9}$$

where the current is given by,

$$J^0 = 2\,\phi' + e\,[(a-1)A_0 + A_1]\,. \tag{2.10}$$

The system given by the Poisson brackets $\{\Omega_1, \Omega_3\}$ constraints is second-class. The time evolution of Ω_3 does not lead to any new constraints but determines the Lagrange multiplier of the Ω_1 constraint. So, the algebra of the constraints is given by a set $\{\Omega_j\}$ which can be determined using the BFFT scheme. In order to simplify this procedure we shall implement the constraints $\Omega_j = 0$ strongly by introducing Dirac brackets [8, 9]. Through the Dirac's procedure we have that

$$\{\Omega_i, \Omega_j\}^D = 0\,, \tag{2.11}$$

and the remaining

$$\{\chi_i(x), \chi_j(y)\}^D = \Delta_{ij}(x,y)\,, \tag{2.12}$$

where we defined $\chi_1 = \Omega_1$ and $\chi_2 = \Omega_3$, from now on $x \equiv x^1$, $y \equiv y^1$ and

$$\Delta_{ij}(x,y) = \begin{pmatrix} 0 & e^2\,(a-1) \\ -e^2(a-1) & 2e^2\partial_x \end{pmatrix} \delta(x-y)\,. \tag{2.13}$$

In order to reduce the second-class system to a first-class one, we begin by extending the phase space including the new fields $\theta_i(x)$ which satisfy the algebra:

$$\{\theta_i(x), \theta_j(y)\}^D = -\,\epsilon_{ij}\,\delta(x-y)\,, \tag{2.14}$$

where $\epsilon^{12} = -\,\epsilon_{12} = +1$.

The first-class $\tilde{\chi}_i$ are now constructed as power series [7, 8, 9]

$$\tilde{\chi}_i = \chi_i + \sum_{n=1}^{\infty} \chi_i^{(n)}, \tag{2.15}$$

where $\chi_i^{(n)}$ are homogeneous polynomials of order n in the auxiliary fields $\theta_i(x)$, to be determined by the requirement that the constraints $\tilde{\chi}_i$ be strongly involutive

$$\{\tilde{\chi}_i(x), \tilde{\chi}_j(y)\}^D = 0. \tag{2.16}$$

The first-order correction for the expression (2.15) can be written as

$$\tilde{\chi}_i = \chi_i + \int \mathrm{d}y \; \sigma_{ij}(x,y) \; \theta_j(y), \tag{2.17}$$

where the quantities $\sigma_{ij}(x,y)$ are implicitly defined by

$$\Delta_{ij}(x,y) = \int \mathrm{d}z \; \mathrm{d}z' \sigma_{ik}(x,z) \, \epsilon_{kl} \, \sigma_{jl}(z',y), \tag{2.18}$$

with $\Delta_{ij}(x,y)$ given by eq. (2.13). By performing the calculations and choosing $\sigma_{ij}(x,y)$ such that $\tilde{\chi}_i$ are linear in the fields $\theta_i(x)$, we obtain,

$$\sigma_{ij}(x,y) = \begin{pmatrix} 1 & 0 \\ \frac{1}{a-1}\partial_x & -e^2(a-1) \end{pmatrix} \delta(x-y). \tag{2.19}$$

Consequently, we get,

$$\begin{aligned} \tilde{\chi}_1 &= \chi_1 + \theta_1(x) \\ \tilde{\chi}_2 &= \chi_2 + \frac{1}{a-1}\partial_1\theta_1 - \, e^2\,(a-1)\theta_2, \end{aligned} \tag{2.20}$$

which are first-class. The above set permit us to compute the extended first-class Hamiltonian,

$$\tilde{H} = \sum_{n=0}^{\infty} H^{(n)}, \tag{2.21}$$

where $H^{(n)} \sim \theta_n$, with the subsidiary condition

$$H^{(0)} \equiv H_c. \tag{2.22}$$

The general expression for the iterated Hamiltonian $H^{(n+1)}$ is given as a recurrence relation as in Refs. [7, 8, 9],

$$H^{(n+1)} \quad = \quad -\frac{1}{n+1}\int \mathrm{d}x \; \mathrm{d}y \; \mathrm{d}z \; \theta_\alpha(x)\, (\omega_{\alpha\beta})^{-1} \, (\sigma_{\beta\gamma})^{-1} \, G_\gamma^{(n)}, \tag{2.23}$$

where

$$G_\gamma^{(n)} = \left\{ \chi_\gamma, H^{(n)} \right\}. \tag{2.24}$$

Here, we mention that $\theta_n = 0$, for $n \geq 2$. Since $(\omega_{\alpha\beta})^{-1}$ and $(\sigma_{\beta\gamma})^{-1}$ are proportional to Dirac delta functions and considering the canonical Hamiltonian, eq. (2.8), we get,

$$\begin{aligned} G_1^{(0)} &= \left\{ \chi_1, H^{(0)} \right\} \\ &= \chi_2 \\ G_2^{(0)} &= \left\{ \chi_2, H^{(0)} \right\} \\ &= e^2\left[(a-1)\partial_1 A_1 - \Pi^1\right]. \end{aligned} \tag{2.25}$$

Performing the shift in the fields

$$\theta_1(x) \quad \longrightarrow \quad e(a-1)\,\theta \tag{2.26}$$

$$\theta_2(x) \quad \longrightarrow \quad \frac{1}{e(a-1)}\Pi_\theta, \tag{2.27}$$

we obtain the first-order correction for the canonical Hamiltonian

$$\begin{aligned} H^{(1)} &= -\int \mathrm{d}x \Big\{ \frac{1}{e(a-1)} \left(\theta' + \Pi_\theta \right) \chi_2 \\ &\quad + \left[(a-1)\partial_1 A_1 - \Pi^1 \right] \theta \Big\}, \end{aligned} \tag{2.28}$$

where $\theta' \equiv \partial_1 \theta$. Following the same steps leading to the first-order corrections we obtain the second-order Hamiltonian (see the Appendix A)

$$H^{(2)} = -\frac{1}{2} \int \mathrm{d}x \left[\frac{1}{a-1} (\Pi_\theta)^2 - \beta (\theta')^2 + e^2 \theta^2 \right], \tag{2.29}$$

with $\beta = (a-1) + (a-1)^{-1}$. Putting together the results from Eqs. (2.8), (2.28) and (2.29), we find the extended Hamiltonian

$$\tilde{H} = H^{(0)} + H^{(1)} + H^{(2)}, \tag{2.30}$$

which is strongly involutive with respect to the constraints $\tilde{\chi}_i(x)$. On the other hand, an inspection of the complete set of constraints reveals that

$$\{ \tilde{\chi}_i, \tilde{\chi}_j \} = 0, \tag{2.31}$$

with $i, j = 1, 2, 3$. These results clearly illuminate the first-class nature of the system. The next step is to calculate the effective action which should be invariant under extended gauge transformations, as we are going to show in the following section.

3 Effective Gauge Invariant Action

Let us obtain the effective action through the BRST-BFV formalism [15]. This method permit us to obtain the effective action in a direct way by including Lagrange multipliers and ghost fields with the corresponding canonical momenta and a gauge fixation function which together with the BRST charge operator generate the terms that lead to the expected gauge invariant action.

Therefore, following the usual BFV prescription and considering the Eqs. (2.28), (2.29), the effective action can be written as,

$$\begin{aligned} S_{eff} &= \int \mathrm{d}^2 x \Big\{ \Pi^0 \dot{A}_0 + \Pi^1 \dot{A}_1 + \Pi_\phi \dot{\phi} + \Pi_\theta \dot{\theta} - \mathcal{H}^{(0)} \\ &\quad + \frac{1}{e(a-1)} (\Pi_\theta + \theta') \chi_2 - e \left[(a-1)\partial_1 A_1 + \Pi^1 \right] \theta \\ &\quad - \frac{1}{2(a-1)} (\Pi_\theta)^2 + \frac{1}{2} \beta (\theta')^2 - \frac{1}{2} e^2 \theta^2 \Big\} \\ &\quad + \int \mathrm{d}^2 x \left[\dot{\lambda}_a p_a + \bar{\mathcal{P}}_a \dot{c}_a + \dot{\bar{c}}_a \mathcal{P}_a + \{ \Psi, Q \} \right], \end{aligned} \tag{3.1}$$

where $(c_a, \bar{\mathcal{P}}_a)$ and $(\mathcal{P}_a, \bar{c_a})$ form a pair of canonical ghost-antighost fields with opposite Grassmanian parity

$$\{ c_a(x), \bar{\mathcal{P}}_b(y) \} = \{ \mathcal{P}_a(x), \bar{c_b}(y) \} = \delta_{ab} \, \delta(x-y), \tag{3.2}$$

while (λ_a, p_a) is a canonical Lagrange multiplier set

$$\{ \lambda_a(x), p_b(y) \} = \delta_{ab} \, \delta(x-y). \tag{3.3}$$

The charge operator Q is defined as

$$Q = c_a \tilde{\chi}_a + p_a \mathcal{P}_a, \tag{3.4}$$

with $\tilde{\chi}_a$ being the first-class constraints, as discussed in the previous section. Finally, the fermion operator Ψ is

$$\Psi = \bar{c}_a \alpha_a + \bar{\mathcal{P}}_a \lambda_a, \tag{3.5}$$

where α_a are the Hermitian gauge-fixing functions. Different choices of the gauge functions α_a can be done in order to obtain the effective action. The partition function is then given by

$$\mathcal{Z} = \int [\mathcal{D}\Sigma]\, e^{i\,S}, \tag{3.6}$$

where the functional integral measure $[\mathcal{D}\Sigma]$ includes all the fields appearing in the action (3.1).

Before going on, we can make the scaling $\alpha_a \to \alpha_a/M$, $p_a \to M p_a$ and $\Sigma_a \to M\Sigma_a$, in such a way that the Jacobian of this transformation is equal to the unity. One can then verifies that in the limit $M \to 0$, the action is independent of ghost and antighost fields.

Now, we can perform the choice of the gauge function and do some of the integrations implied in $[\mathcal{D}\Sigma]$. To this end, we choose

$$\alpha_1(x) = \Pi_\phi - \phi' + \dot{\lambda}_1 \tag{3.7}$$

$$\alpha_2(x) = \Pi_\theta + \theta' + e(a-1)A_0 + eA_1 + \dot{\lambda}_2. \tag{3.8}$$

Since the first-class constraints are

$$\tilde{\chi_1} \;=\; \chi_1 + e\,(a-1)\,\theta \tag{3.9}$$

$$\tilde{\chi_2} \;=\; \chi_2 + e\,(\theta' - \Pi_\theta), \tag{3.10}$$

where $\chi_1 \equiv \Omega_1$ and $\chi_2 \equiv \Omega_3$, given by Eqs. (2.6), (2.7) and (2.9), we can compute the Poisson bracket

$$\begin{aligned} \{\Psi, Q\} \;=\; & -[\Pi_\phi - \phi' + \dot{\lambda}_1]p_1 \\ & -[\Pi_\theta + \theta' + e\,(a-1)\,A_0 \,+\, e\,A_1 \,+\, \dot{\lambda}_2]\,p_2 \\ & -[\Pi_0 + e\,(a-1)\,\theta]\lambda_1 \\ & -[\chi_2 + e\,(\theta' - \Pi_\theta)]\lambda_2. \end{aligned} \tag{3.11}$$

The dynamical terms $\dot{\lambda}_a p_a$ which appear in Eq. (3.11) are cancelled by the similar ones in the original action. After integrations over the fields (p_1, p_2) and (λ_1, λ_2), we arrive at the delta functionals $\delta(\Pi_\theta + \theta' + e(a-1)A_0 + eA_1)$, $\delta(\Pi_\phi - \phi')$, $\delta(\Pi_0 + e(a-1)\theta)$ and $\delta(\chi_2 + e\,\theta' - e\Pi_\theta)$ in the partition function (3.6). Performing the integrations over (Π_ϕ, Π_θ) and (Π^0, Π^1) and using the fact that the Hamiltonian $H^{(0)}$ is quadratic in the field Π^1, we get the gauge invariant (GI) effective action

$$S^{GI}_{eff}[\phi, \theta, A_\mu] = \check{S}[\phi, A_\mu] + S[\theta, A_\mu]\,, \tag{3.12}$$

where $\check{S}[\phi, A_\mu]$ is the action corresponding to the Lagrangian (2.3),

$$\begin{aligned} \check{S}[\phi, A_\mu] \;=\; & \int \mathrm{d}^2x \Big\{ \dot{\phi}\,\phi' - (\,\phi')^2 + 2e\,\phi'(A_0 - A_1) - \frac{1}{2}\,e^2\,(A_0 - A_1)^2 \\ & + \frac{1}{2}\,e^2\,a[(A_0)^2 - (A_1)^2] \Big\}\,, \end{aligned} \tag{3.13}$$

and $S[\theta, A_\mu]$ is given by,

$$S[\theta, A_\mu] = S_{wz}[\theta, A_\mu] - \frac{1}{2}\frac{e^2}{a-1} \int d^2x[(a-1)A_0 + A_1]^2\,, \tag{3.14}$$

with the WZ action given by,

$$S_{wz}[\theta, A_\mu] = \int \mathrm{d}^2x \Big\{ -\dot{\theta}\,\theta' - \frac{1}{2}\beta(\,\theta')^2 + \,e\,\beta A_1\,\theta' \,+\, 2\,e\,A_0\,\theta' \Big\}\,. \tag{3.15}$$

The effective GI action can be rewritten as,

$$S_{eff}^{GI}[\phi,\theta,A_\mu] = S_{eff}^{GNI}[\phi,A_\mu] \,+\, S_{_{WZ}}[\theta,A_\mu]\,, \tag{3.16}$$

where the GNI action is given by,

$$S_{eff}^{GNI}[\phi,A_\mu] = \int \mathrm{d}^2x \left\{ \dot\phi\,\phi' - (\,\phi')^2 \,+\, 2e\,\phi'(A_0 - A_1) \,-\, \frac{1}{2}\,e^2\,(\beta+2)\,(A_1)^2 \right\} \tag{3.17}$$

The effective action (3.16) is invariant under extended gauge transformations,

$$^g\phi = \phi + g\,, \tag{3.18}$$

$$^g\theta = \theta - g\,, \tag{3.19}$$

$$^gA_\mu = A_\mu - \frac{1}{e}\,\partial_\mu g\,. \tag{3.20}$$

This could be achieved with the use of the first-class constraints $\tilde\chi_1$, $\tilde\chi_2$, representing the gauge symmetry of the model, introduced by the use of the BFFT method as discussed in the previous section. The generators of the symmetry transformations can be written as [30]

$$G = \int \mathrm{d}x\,(a_1\tilde\chi_1 + a_2\tilde\chi_2)\,, \tag{3.21}$$

where the coefficients a_j are determined through the relations $\{\chi_1, H_c\} = a_1\chi_1$ and $\{\chi_2, H_c\} = a_2\chi_2$. From Eq. (2.9), we obtain

$$\begin{aligned} \delta A_1 &= \{A_1, G\}\epsilon \\ &= -\frac{1}{e}\partial_1\epsilon\,, \end{aligned} \tag{3.22}$$

and similarly $\delta\phi = \epsilon = -\delta\theta$, where $\epsilon = \epsilon(x)$ is a gauge parameter.

4 Operator Gauge Transformation and the WZ Action

As was stressed in Ref. [31] for the standard chiral QED_2 and in [32] for the chiral QCD_2, the WZ action can be obtained via an operator-valued gauge transformation of its GNI effective quantum action. The resulting extended GI theory is isomorphic to original GNI theory. The isomorphism between these two formulations is valid in an arbitrary gauge. In what follows we show that in the minimal chiral model the GI formulation can also be obtained by performing a gauge transformation on the GNI formulation. To begin with, let us consider the q-number gauge transformation

$$\phi \,\rightarrow\, {}^\theta\phi = \phi + \theta\,, \tag{3.23}$$

$$A_\mu \,\rightarrow\, {}^\theta A_\mu = A_\mu - \frac{1}{e}\,\partial_\mu\theta\,, \tag{3.24}$$

acting on the effective GNI action given by (3.17),

$$\begin{aligned} {}^\theta S_{eff}^{GNI}[\phi,A_\mu] \;&=\; \int \mathrm{d}^2x \Big\{ (\,\dot\phi + \dot\theta)(\,\phi' + \theta') - (\,\phi' + \theta')^2 \\ &+2e(\,\phi' + \theta')\Big(A_0 - A_1 - \frac{1}{e}(\,\dot\theta - \theta')\Big) \\ &- \frac{1}{2}\,e^2(\beta+2)\,(A_1 - \frac{1}{e}\,\theta')^2\,. \end{aligned} \tag{3.25}$$

The gauge transformed action (3.25) is manifest invariant under extended gauge transformations (3.18),(3.19), (3.20), and is identical to the extended GI effective action given by (3.12),

$$
{}^{\theta}S^{GNI}_{eff}[\phi, A_\mu] = S^{GI}_{eff}[\phi, A_\mu,\, \theta] = S^{GNI}_{eff}[\phi, A_\mu] \,+\, S_{WZ}[\theta, A_\mu]\,. \tag{3.26}
$$

The WZ action (3.15) can also be obtained by performing a q-number gauge transformation on the bosonized action corresponding to the original GNI Lagrangian (2.3),

$$
\begin{aligned}
\check{S}[\phi, A_\mu] \;\; &= \;\; \int d^2x \Big\{ \,\dot\phi\, \phi' - (\,\phi')^2 + 2\,e\,\phi'(A_0 - A_1) \\
&\qquad -\frac{1}{2}\,e^2\,(A_0 - A_1)^2 + \frac{1}{2}\,a\;e^2\,\Big((A_0)^2 - (A_1)^2\Big)\Big\}\,.
\end{aligned} \tag{3.27}
$$

In this way, using (3.23)-(3.24), we obtain

$$
{}^{\theta}\check{S}[\phi, A_\mu] = \check{S}[\phi, \theta, A_\mu] = \check{S}[\phi, A_\mu] + \check{S}_{WZ}[\theta, A_\mu]\,, \tag{3.28}
$$

where the WZ action is now given by,

$$
\begin{aligned}
\check{S}_{WZ}[\theta, A_\mu] \;\; &= \;\; \int d^2x \Big\{ \frac{1}{2}(a-1)\,(\,\dot\theta)^2 - \frac{1}{2}(a-1)(\,\theta')^2 \\
&\qquad + e\,A_0\,\Big(\,\theta' - (a-1)\;\dot\theta\Big) + \,e\,A_1\,\Big((a-1)\,\theta' - \,\dot\theta\Big)\Big\}\,.
\end{aligned} \tag{3.29}
$$

The action (3.29) is the usual WZ action obtained for the standard chiral model. In this case, the canonical momentum associated with the field θ is given by,

$$
\Pi_\theta = (\,a\, - 1)\;\dot\theta - \,e\,[(a-1)A_0 + A_1]\,. \tag{3.30}
$$

The effective GI action obtained in the previous section using the BRST-BFV formalism, is gauge fixed by the gauge conditions (3.7)-(3.8). In order to map the WZ action (3.29) into (3.15), we must impose on (3.29) the following condition

$$
\theta' + (a-1)\;\dot\theta \approx 0\,. \tag{3.31}
$$

The condition (3.31) play the role of the delta function $\delta(\Pi_\theta + \,\theta' + \,e\,(a-1)A_0 + A_1)$ that appears in the integrations over the fields (p_1, p_2) and (λ_1, λ_2) performed in the previous section to obtain the effective action. Under the condition (3.31) the canonical momentum (3.30) is mapped into

$$
\Pi_\theta = -\,\theta' - \,e\,[(a-1)A_0 + A_1]\,. \tag{3.32}
$$

Indeed, rewriting the standard WZ action (3.29) as,

$$
\begin{aligned}
\check{S}_{WZ}[\theta, A_\mu] \;\; &= \;\; \int d^2x\,\Big\{(a-1)(\,\dot\theta)^2 - \frac{1}{2}(a-1)[(\,\dot\theta)^2 + (\,\theta')^2]\Big\} \\
&\qquad + e\,A_0\,\Big(\,\theta' - (a-1)\,\dot\theta\Big) + \,e\,A_1\,\Big((a-1)\,\theta' - \,\dot\theta\Big)\Big\}\,,
\end{aligned} \tag{3.33}
$$

and using (3.31), such that

$$
\begin{aligned}
&(a-1)\dot\theta\,\theta - \tfrac{1}{2}(a-1)[(\,\dot\theta)^2 + (\,\theta')^2] \\
= &-\dot\theta\,\theta' - \tfrac{1}{2}(a-1)[\tfrac{1}{(a-1)^2}(\,\theta')^2 + (\,\theta')^2]\,,
\end{aligned} \tag{3.34}
$$

we obtain from (3.29),

$$
\begin{aligned}
\check{S}_{WZ}[\theta, A_\mu] \;\; \rightarrow \;\; & S_{WZ}[\theta, A_\mu] \\
&= \int d^2x \Big\{ -\,\dot\theta\,\theta' - \frac{1}{2}\beta(\,\theta')^2 + \,e\,\beta\,A_1\,\theta' \,+\, 2\,e\,A_0\,\theta' \Big\}\,,
\end{aligned} \tag{3.35}
$$

in agreement with the minimal WZ action, eq. (3.15), obtained in the previous section.

5 Concluding Remarks

We have obtained the extended gauge invariant version of the minimal chiral Schwinger model (for $a \neq 1$) by using the BFFT method. Consequently, this gauge invariance might suggest that this version of the model (possibly with a convenient choice of the Jackiw-Rajaraman regularization parameter a) has some correspondence with a bonafide gauge model, such as the QED_2, which is naturally gauge invariant, or in other words, is not gauge anomalous since in this case the anomaly resides in the axial current.

Indeed, it has been argued by Kye *et al.* [27] that in the case $a = 2$ there is a correspondence between the GI formulation of the minimal chiral model and the QED_2 (see also [29]). From our point of view, this limit in the standard chiral models as well as in the minimal chiral models, does not physically represent the (vector) Schwinger model, in contrast to the equivalence advocated in the literature [27, 29].

In Ref. [31] the fermionization of the standard WZ Lagrangian has been performed for general Jackiw-Rajaraman parameter $a > 1$. The fermionized version of the WZ Lagrangian can be written as a Thirring model plus a coupling of the gauge field with the axial and vector currents. For the special value $a = 2$ the Thirring coupling vanishes and the total Lagrangian of the GI version can be related to a model exhibiting some resemblance with the QED_2. However, this mapping only has a formal character since it only can be performed in the Lagrangian level. From the functional integral approach, this mapping only can be performed into the partition function. These correspondences cannot be established for the generating functionals, which implies that there is no isomorphism between the corresponding Hilbert space of states. In this way, the models cannot be considered as being equivalent. As stressed in Refs. [20, 21, 32], from the operator point of view, the claimed equivalence is a consequence of an improper factorization of the Hilbert space that implies the choice of a field operator that does not belong to the intrinsic field algebra to represent the fermionic content of the model. Contrary to the alleged equivalence, we also mention two general properties of two-dimensional anomalous gauge theories: *i*) the anomalous models do not exhibit the violation of the asymptotic factorization property (cluster decomposition) and thus there is no need of a θ-vacuum parametrization [20, 21, 32]; *ii*) the models exhibit a peculiar feature which allows two isomorphic formulations: the GNI and GI formulations [20, 21]. The suggested equivalence of the chiral model for $a = 2$ and the vector model can not be established if we consider the Hilbert space in which the intrinsic field algebra of the model is represented [16, 20, 21].

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Appendix A

Here, we are going to calculate $H^{(2)}$, Eq. (2.29). First of all, we give some details of the calculation of $G_1^{(1)}$ and $G_2^{(1)}$, which are necessary for determining $H^{(2)}$. From the definition of $G_\gamma^{(n)}$, Eq. (2.24), we have

$$\begin{aligned} G_1^{(1)} &= \left\{\chi_1, H^{(1)}\right\} \\ &= \theta_2(x)\left\{\chi_1, -\chi_2\right\}(\partial_1\theta_1(x))\ \frac{1}{e^2(a-1)^2}\left\{\chi_1,\, e^2\,(a-1)A_0\right\} \\ &= -\frac{1}{a-1}\left(\partial_1\theta_1(x)\right) - e^2(a-1)\theta_2(x). \end{aligned} \tag{A.1}$$

Also,

$$\begin{aligned} G_2^{(1)} &= \left\{\chi_2, H^{(1)}\right\} \\ &= -\left[\partial_1\theta_1(x)\right]\frac{1}{e^2(a-1)}\left\{\chi_2,\chi_2\right\} - \theta_2(x)\left\{\chi_2,\chi_2\right\} \\ &\quad -\theta_1(x)\frac{1}{e^2(a-1)}\left\{\chi_2, e^2\left[(a-1)\partial_1 A_1 + \Pi^1\right]\right\} \\ &= -\frac{2}{(a-1)^2}\left(\partial_1\theta_1(x)\right)\partial_1 - \theta_1(x)(\partial_1)^2 \\ &\quad -\frac{e^2}{a-1}\theta_1(x) - 2e^2\theta_2(x)\partial_1. \end{aligned} \tag{A.2}$$

Now, let us calculate $H^{(2)}$

$$
\begin{aligned}
H^{(2)} &= -\frac{1}{2}\int \mathrm{d}x\;\theta_1(x)\,\epsilon_{12}\,\sigma^{21}G_1^{(1)} \\
&\quad -\frac{1}{2}\int \mathrm{d}x\;\theta_1(x)\,\epsilon_{12}\,\sigma^{22}G_2^{(1)} \\
&\quad -\frac{1}{2}\int \mathrm{d}x\;\theta_2(x)\,\epsilon_{21}\,\sigma^{11}G_1^{(1)}
\end{aligned}
\tag{A.3}
$$

where $\sigma^{ij} = (\sigma_{ij})^{-1}$, with σ_{ij} given by Eq. (2.19), so that

$$
(\sigma_{ij})^{-1} = \begin{pmatrix} 1 & 0 \\ \frac{1}{e^2(a-1)^2}\partial_x & -\frac{1}{e^2(a-1)} \end{pmatrix}\delta(x-y). \tag{A.4}
$$

Then, we have

$$
\begin{aligned}
H^{(2)} &= -\frac{1}{2}\int \mathrm{d}x\;\theta_1(x)\frac{1}{e^2(a-1)^2}\partial_x\Big[\frac{1}{a-1}(\partial_1\theta_1(x)) + (a-1)e^2\theta_2(x)\Big] \\
&+ \int \mathrm{d}x\;\theta_1(x)\frac{1}{e^2(a-1)}\Big[\frac{1}{(a-1)^2}(\partial_1\theta_1(x))\partial_x \\
&\qquad + \frac{1}{2}\theta_1(x)(\partial_x)^2 + \frac{1}{2}\frac{1}{(a-1)e^2}\theta_1(x) + e^2\,\theta_2(x)\partial_x\Big] \\
&+ \frac{1}{2}\int \mathrm{d}x\;\theta_2(x)\Big[\frac{1}{a-1}(\partial_1\theta_1(x)) + (a-1)e^2\theta_2(x)\Big]
\end{aligned}
\tag{A.5}
$$

Finally, after some algebra and making the shift given by Eqs. (2.26), (2.27), one arrives at $H^{(2)}$, Eq. (2.29).

References

[1] P. A. M. Dirac, Can. J. Phys. 2 (1950) 129; "Lectures on Quantum Mechanics", Yeshiva University, New York, 1964.

[2] M. Henneaux and C. Teitelboim, "Quantization of Gauge Systems", Princeton University Press, Princeton, 1992.

[3] I. A. Batalin and E. S. Fradkin, Phys. Lett. **B 180**, 157 (1986); *ibid.* Nucl. Phys. **B279**, 514 (1987).

[4] I. A. Batalin, E. S. Fradkin and T. E. Fradkina, Nucl. Phys. **B314**, 158 (1989); *ibid.* **B323**, 734 (1989).

[5] I. A. Batalin and I. V. Tyutin, Int. J. Mod. Phys. A **6**, 3255 (1991).

[6] R. Amorim and J. Barcelos-Neto, Phys. Lett. **B 333**, 413 (1994); *ibid.* Phys. Rev. **D 53**, 7129 (1996).

[7] T. Fujikawa, Y. Igarashi and J. Kubo, Nucl. Phys. **B341**, 695 (1990).

[8] R. Banerjee and J. Barcelos-Neto, Nucl. Phys. **B499** (1997) 453;

[9] Y. W. Kim and K. D. Rothe, Nucl. Phys. **B 511**, 510 (1998).

[10] N. Banerjee, R. Banerjee and S. Gosh, Phys. Rev. **D 49**, 1996 (1994).

[11] J. Barcelos-Neto and W. Oliveira, Phys. Rev. **D 56** (1997) 2257.

[12] W. T. Kim, Y.-W. Kim, M.-I. Park, Y.-J. Park, S. J. Yoon, J. Phys. G **23**, 325 (1997).

[13] M. I. Park, Y. J. Park and S. Yoon, J. Phys. G **24**, 2179 (1998).

[14] C. P. Natividade and H. Boschi-Filho, Phys. Rev. **D 62**, 025016-1 (2000).

[15] M. Henneaux, Phys. Rep. **126**, 1 (1985).

[16] E. Abdalla, M. C. B. Abdalla and K. D. Rothe, "Non-perturbative methods in 2 dimensional quantum field theory", World Scientific, Singapore, 1991.

[17] J. Schwinger, Phys. Rev. **128**, 2425 (1962).

[18] J. H. Lowestein and J. A. Swieca, Ann. Phys. (N.Y.) **65**, 172 (1971).

[19] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. **54**, 1219 (1985).

[20] C. G. Carvalhães, L. V. Belvedere, C. P. Natividade and H. Boschi-Filho, Ann. Phys. (N.Y.) **258**, 210 (1997).

[21] C. G. Carvalhaes, L. V. Belvedere, R. L. P. G. do Amaral and N. A. Lemos, Ann. Phys. **269**, 1 (1998).

[22] W. Siegel, Nucl. Phys. **B238**, 307 (1984).

[23] R. Floreanini and R. Jackiw, Phys. Rev. Lett. **59**, 1872 (1987).

[24] K. Harada, Phys. Rev. Lett. **64**, 139 (1990).

[25] K. Harada, Phys. Rev. **D 42** (1990) 4170.

[26] J.K. Kim, W.T. Kim and W.H. Kye, Phys. Rev. **D 43** (1991) 626; *ibid.* **45** (1992) 717 (E).

[27] W.-H. Kye, W.-T. Kim and J.-K. Kim, Phys. Lett. **B 268**, 59 (1991).

[28] A. de Souza Dutra, Phys. Lett. **B 286**, 285 (1992).

[29] C. E. M. Wagner and M. Carena, Int. J. Mod. Phys. **A 6**, 243 (1991).

[30] Carlos A. P. Galvão and J. B. Boechat, J. Math. Phys. **31**, 448 (1990).

[31] L. V. Belvedere and K. D. Rothe, Modern Phys. Lett. **A 10** (1995) 207.

[32] L. V. Belvedere, R. L. P. G. Amaral and K. D. Rothe, Int. J. Mod. Phys. **A 14**, (1999) 1163.